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NATURAL INFLATION*

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ABSTRACT

A pseudo-Nambu-Goldstone boson, with a potential of the form $V(\phi) = \Lambda^4[1 \pm \cos(\phi/f)]$, can naturally give rise to an epoch of inflation in the early universe. Successful inflation can be achieved if $f \sim m_{pl}$ and $\Lambda \sim m_{GUT}$. Such mass scales arise in particle physics models with a gauge group that becomes strongly interacting at the GUT scale, *e.g.*, as is expected to happen in the hidden sector of superstring theories. The density fluctuation spectrum is a non-scale-invariant power law, with extra power on large scales.

In recent years, the inflationary universe has been in a state of theoretical limbo: it is a beautiful idea in search of a compelling model. The idea is simple[1]: if the early universe undergoes an epoch of exponential de Sitter expansion during which the scale factor increases by a factor of at least e^{60}, then a small causally connected region grows to a sufficiently large size to explain the observed homogeneity and isotropy of the universe, to dilute any overdensity of magnetic monopoles, and to flatten the spatial hypersurfaces, $\Omega \equiv 8\pi G\rho/3H^2 \rightarrow 1$. As a bonus, quantum fluctuations during inflation can causally generate the large-scale density fluctuations required for galaxy formation.

During the inflationary epoch, the energy density of the universe is dominated by the (nearly constant) potential energy density $V(\phi)$ associated with a slowly rolling scalar field ϕ, the ***inflaton*** [2]. To satisfy microwave background anisotropy limits [3] on the generation of density fluctuations, the potential of the inflaton must be very flat. Consequently, ϕ must be extremely weakly self-coupled, with effective quartic self-coupling constant $\lambda_\phi < 10^{-12} - 10^{-14}$.

Thus, density fluctuations in inflation are a blessing to astronomers but a curse on particle physicists: although a large number of inflation models have been proposed [4], none of them is aesthetically compelling from a particle physics standpoint. In some cases, the smallness of λ_ϕ is protected against radiative corrections by a symmetry, *e.g.*, supersymmetry. However, the small coupling, while stable (technically natural), is itself unexplained, and is postulated solely in order to generate successful inflation. In recent

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years, it has become customary to decouple the inflaton completely from particle physics models, to specify an 'inflaton sector' with the requisite properties, with little or no regard for its physical origin. It would be preferable if the small coupling of the inflaton arose dynamically in particle physics models which are *strongly* natural, *i.e.*, which have no small numbers in the Lagrangian.

An example of the kind of thing we want, namely, a scalar field with naturally small self-coupling, is provided by the axion [5]. In axion models, a global $U(1)$ symmetry is spontaneously broken at some large mass scale f, through the vacuum expectation value of a complex scalar field, $\langle\Phi\rangle = f\ \exp(ia/f)$. (In this case, Φ has the familiar Mexican-hat potential, and the vacuum is a circle of radius f.) At energies below the scale f, the only relevant degree of freedom is the massless axion field a, the angular Nambu-Goldstone mode around the bottom of the Φ potential. However, at a much lower scale, the symmetry is explicitly broken by loop corrections. For example, the QCD axion obtains a mass from non-perturbative gluon configurations (instantons) through the chiral anomaly. When QCD becomes strong at the scale $\Lambda_{QCD} \sim 100$ MeV, instanton effects give rise to a periodic potential of height $\sim \Lambda^4_{QCD}$ for the axion. In 'invisible' axion models [6] with canonical Peccei-Quinn scale $f_{PQ} \sim 10^{12}$ GeV, the resulting axion self-coupling is $\lambda_a \sim (\Lambda_{QCD}/f_{PQ})^4 \sim 10^{-52}$. This simply reflects the hierarchy between the QCD and Peccei-Quinn scales, which arises from the slow logarithmic running of α_{QCD}. Since the global symmetry is restored as $\Lambda \rightarrow 0$, the flatness of the axion potential is natural.

Pseudo-Nambu-Goldstone bosons (PNGBs) like the axion are ubiquitous in particle physics models: they arise whenever a global symmetry is spontaneously broken. We therefore choose them as our candidate for the inflaton: we assume a global symmetry is spontaneously broken at a scale f, with soft explicit symmetry breaking at a lower scale Λ; these two scales completely characterize the model and will be specified by the requirements of successful inflation. The resulting PNGB potential is generally of the form

$$V(\phi) = \Lambda^4[1 + \cos(\phi/f)] \,. \tag{1}$$

so the potential, of height $2\Lambda^4$, has a unique minimum at $\phi = \pi f$. As we will see below, for $f \sim m_{pl} \sim 10^{19}$ GeV and $\Lambda \sim m_{GUT} \sim 10^{15}$ GeV, the PNGB field ϕ can drive inflation [7]. (Note that in this case, the effective quartic coupling is $\lambda_\phi \sim (\Lambda/f)^4 \sim 10^{-13}$, as required.) These mass scales arise naturally in particle physics models. For example, in the hidden sector of superstring theories, if a non-Abelian subgroup of E_8 remains unbroken, the running gauge coupling can become strong at the GUT scale; indeed, it is hoped that the resulting gaugino condensation may play a role in breaking supersymmetry [8]. In this case, the role of the PNGB inflaton could be played by the "model-independent axion" [9].

For temperatures $T \lesssim f$, the global symmetry is spontaneously broken. Since ϕ thermally decouples at a temperature $T \sim f^2/m_{pl} \sim f$, we assume it is initially laid down at random between 0 and $2\pi f$ in different causally connected regions. Within each Hubble volume, the evolution of the field is described by

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma\dot{\phi} + V'(\phi) = 0 \,, \tag{2}$$

where Γ is the decay width of the inflaton. In the temperature range $\Lambda \lesssim T \lesssim f$, the potential $V(\phi)$ is dynamically irrelevant, because the forcing term $V'(\phi)$ is negligible compared to the Hubble-damping term. (In addition, for axion models, $\Lambda \to 0$ as $T/\Lambda \to \infty$ due to the high-temperature suppression of instantons.) Thus, in this temperature range, aside from the smoothing of spatial gradients in ϕ, the field does not evolve. Finally, for $T \lesssim \Lambda$, in regions of the universe with ϕ initially near the top of the potential, the field starts to roll slowly down the hill toward the minimum. In those regions, the energy density of the universe is quickly dominated by the vacuum contribution ($V(\phi) \simeq 2\Lambda^4 \gtrsim \rho_{rad} \sim T^4$), and the universe expands exponentially. Since the initial conditions for ϕ are random, our model is closest in spirit to the chaotic inflationary scenario [10].

To successfully solve the cosmological puzzles of the standard cosmology, an inflationary model must satisfy a variety of constraints.

1) *Slow-Rolling Regime.* The field is said to be slowly rolling (SR) when its motion is overdamped, *i.e.*, $\ddot{\phi} << 3H\dot{\phi}$, and two conditions are met:

$$|V''(\phi)| \lesssim 9H^2 \ , \text{ i.e.,} \quad \sqrt{\frac{2\,|\cos(\phi/f)|}{1+\cos(\phi/f)}} \lesssim \frac{\sqrt{48\pi}f}{m_{pl}} \tag{3}$$

and

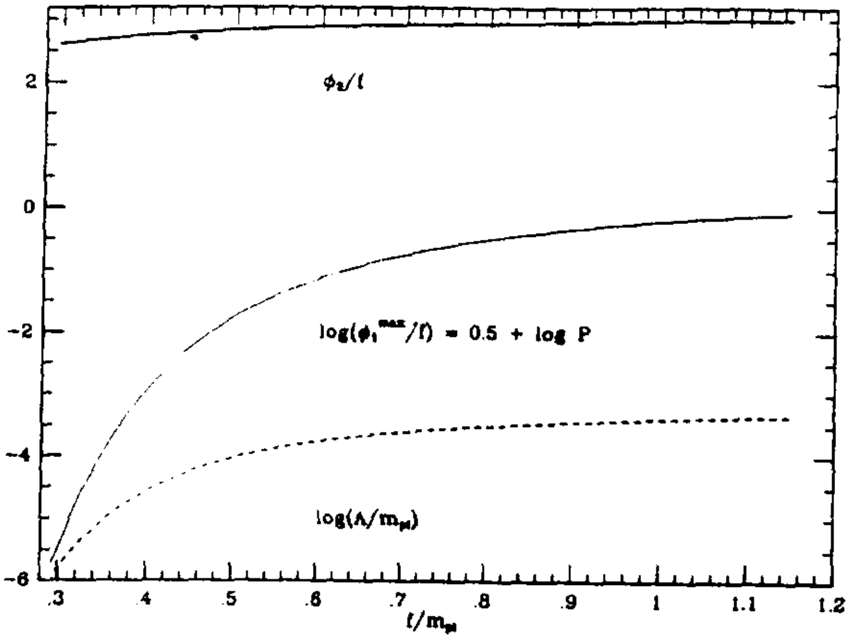
$$\left|\frac{V'(\phi)m_{pl}}{V(\phi)}\right| \lesssim \sqrt{48\pi} \ , \text{ i.e.,} \quad \frac{\sin(\phi/f)}{1+\cos(\phi/f)} \lesssim \frac{\sqrt{48\pi}f}{m_{pl}} \ . \tag{4}$$

From Eqns. (3) and (4) the existence of a broad SR regime requires $f \geq m_{pl}/\sqrt{48\pi}$ (required below for other reasons). The SR regime ends when ϕ reaches a value ϕ_2, at which one of the inequalities (3) or (4) is violated. For example, for $f = m_{pl}$, $\phi_2/f = 2.98$ (near the potential minimum), while for $f = m_{pl}/\sqrt{24\pi}$, $\phi_2/f = 1.9$. As f grows, ϕ_2/f approaches π. (Here and below, we assume inflation begins at a field value $0 < \phi_1/f < \pi$; since the potential is symmetric about its minimum, we can just as easily consider the case $\pi < \phi_1/f < 2\pi$.)

2) *Sufficient inflation.* We demand that the scale factor of the universe inflates by at least 60 e-foldings during the SR regime,

$$\begin{aligned} N_e(\phi_1,\phi_2,f) \equiv \ln(R_2/R_1) &= \int_{t_1}^{t_2} H\,dt = \frac{-8\pi}{m_{pl}{}^2}\int_{\phi_1}^{\phi_2}\frac{V(\phi)}{V'(\phi)}d\phi \\ &= \frac{16\pi f^2}{m_{pl}{}^2}\ln\left[\frac{\sin(\phi_2/2f)}{\sin(\phi_1/2f)}\right] \geq 60 \ . \end{aligned} \tag{5}$$

Using Eqns. (3) and (4) to determine ϕ_2 as a function of f, the constraint (5) determines the maximum value (ϕ_1^{max}) of ϕ_1 consistent with sufficient inflation. The fraction of the universe with $\phi_1 \in [0, \phi_1^{max}]$ will inflate sufficiently. If we assume that ϕ_1 is randomly distributed between 0 and πf from one horizon volume to another, the probability of being in such a region is $P = \phi_1^{max}/\pi f$. For example, for $f = 3m_{pl}$, m_{pl}, $m_{pl/2}$, and $m_{pl}/\sqrt{24\pi}$, the probability $P = 0.7$, 0.2, 3×10^{-3}, and 3×10^{-41}. The fraction of the universe that inflates sufficiently drops precipitously with decreasing f, but is large for f near m_{pl}. This is shown in Fig. 1, which displays $\log(\phi_1^{max}/f) = 0.5 + \log P$ and ϕ_2/f.



3) *Density Fluctuations.* Inflationary models generate density fluctuations [11] with amplitude at horizon crossing $\delta\rho/\rho \simeq 0.1H^2/\dot{\phi}$, where the right hand side is evaluated when the fluctuation crossed outside the horizon during inflation. Fluctuations on observable scales are produced 60 - 50 e-foldings before the end of inflation. The largest amplitude perturbations are produced 60 e-foldings before the end of inflation,

$$\frac{\delta\rho}{\rho} \simeq \frac{0.3\Lambda^2 f}{m_{pl}^3}\left(\frac{8\pi}{3}\right)^{3/2}\frac{[1+\cos(\phi_1^{max}/f)]^{3/2}}{\sin(\phi_1^{max}/f)}\,. \tag{6}$$

Constraints on the anisotropy of the microwave background [3] require $\delta\rho/\rho \leq 5\times 10^{-5}$, *i.e.*, $\Lambda \leq 2\times 10^{16}$ GeV for $f = m_{pl}$ and $\Lambda \leq 3\times 10^{15}$ GeV for $f = m_{pl}/2$. This bound on Λ as a function of f is also shown in the figure. Thus, to generate the fluctuations responsible for large-scale structure, Λ should be comparable to the GUT scale, and the inflaton mass $m_\phi = \Lambda^2/f \sim 10^{11} - 10^{12}$ GeV.

In this model, the fluctuations deviate from a scale-invariant spectrum: the amplitude at horizon-crossing grows with mass scale M as $\delta\rho/\rho \sim M^{m_{pl}^2/48\pi f^2}$. Thus, the primordial power spectrum (at fixed time) is a power law, $|\delta_k|^2 \sim k^n$, with spectral index $n = 1 - (m_{pl}^2/8\pi f^2)$. The extra power on large scales (compared to the scale-invariant $n=1$ spectrum) may have important implications for large-scale structure [12].

4) *Reheating.* At the end of the SR regime, the field ϕ oscillates about the minimum of the potential, and gives rise to particle and entropy production. The decay of ϕ into fermions and gauge bosons reheats the universe to a temperature $T_{RH} = (45/4\pi^3 g_*)^{1/4}\sqrt{\Gamma m_{pl}}$, where g_* is the number of relativistic degrees of freedom. On dimensional grounds, the decay rate is $\Gamma \simeq g^2 m_\phi{}^3/f^2 = g^2\Lambda^6/f^5$, where g is an effective coupling constant. (For example, in the original axion model [5,6], $g \propto \alpha_{EM}$ for

two-photon decay, and $g^2 \propto (m_\psi/m_\phi)^2$ for decays to light fermions ψ.) For $f = m_{pl}$ and $g_* = 10^3$, we find $T_{RH} = 10^8 g$ GeV, too low for conventional GUT baryogenesis, but high enough if baryogenesis takes place at the electroweak scale. Alternatively, the baryon asymmetry can be produced directly during reheating through baryon-violating decays of ϕ or its decay products. The resulting baryon-to-entropy ratio is $n_B/s \simeq \epsilon T_{RH}/m_\phi \sim \epsilon g \Lambda/f \sim 10^{-4}\epsilon g$, where ϵ is the CP-violating parameter; provided $\epsilon g \gtrsim 10^{-6}$, the observed asymmetry can be generated.

In conclusion, a pseudo-Nambu-Goldstone boson, *e.g.,* a heavy (non-QCD) axion, with a potential that arises naturally from particle physics models, can lead to successful inflation if the global symmetry breaking scale $f \simeq m_{pl}$ and $\Lambda \simeq m_{GUT}$.

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